Optimization of the Detection Process Timing in Molecular Communication Systems With Flow

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***Abstract*—In this paper, we analyze the error performance of a diffusive molecular communication system in the presence of flow. Our aim is to enhance both the timing and number of the receiver's observations used in the detection process. To this end, closed-form expressions for the bit error rate are presented and minimized in terms of the optimal sampling time and the ideal number of samples that should be collected within the observation interval.**

***Index Terms*—Amplitude detection, molecular communication, sampling time, single threshold detection.**

I. Introduction

Nanotechnology is a scientific field with achievements that have given rise to versatile new needs, including the communication between nanoscale devices. Inspired by biological mechanisms, molecular communication (MC) utilizes molecules for the transfer of information between nanomachines [1], [2]. In this case, information carrying molecules diffuse freely and randomly in the communication medium and arrive at the receiver in a probabilistic manner. Besides, flow (or drift) is likely to be present in any environment where MC is deployed [3]. To this end, several works in the literature, dealing with diffusion-based MC systems, consider motion models where the drift is also incorporated [3]–[6].

One of the challenges in MC is the design of reliable receivers of the least possible complexity. Depending on the requirements and the constraints of the considered application, different types of receivers can be utilized. Among them, very popular are those that do not react with the molecules that reach their surface, namely the *passive receivers*. Passive receivers can detect the information by counting the number of received molecules in predefined time instances, which is referred to as *amplitude detection with sampling* [3].

In this paper, we extend the analysis presented in [7] by studying a diffusion-based MC system with the consideration of flow in the motion model of the molecules. Particularly, we assume that molecules propagate via Brownian motion with a net drift from the transmitter to the receiver, generally called positive drift [5]. This system model is very common in diffusion-based MC systems with healthcare applications inside one of the blood vessels of a human body [6]. By utilizing closed-form expressions for the bit error rate, we specify the optimal time, when the observation process should start and finish, as well as the ideal number of samples which should be collected within the observation interval, that minimize the error probability.

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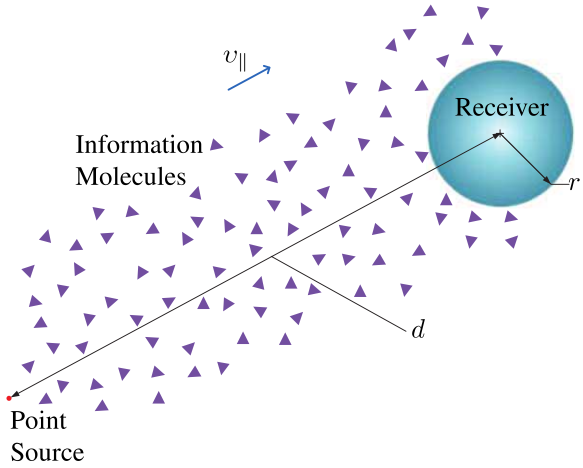

Fig. 1: Molecular communication system via diffusion with flow (or drift).

II. System Model

The MC system under consideration consists of a transmitter-receiver pair placed in a three-dimensional and stationary environment, which is unbounded in all dimensions and filled with a fluid of uniform temperature and constant viscosity. The transmitter is a point source, i.e., zero-dimensional, where the information is modulated via the number of molecules of a certain type that are emitted instantaneously at the beginning of the symbol duration T. In this paper, we assume ON/OFF keying modulation, which implies that emitting M molecules corresponds to bit-1 and no emission (i.e., zero molecules) to bit-0, while both cases are considered equiprobable.

The information molecules diffuse freely towards all directions after their emission, but also have a constant displacement due to flow. We assume a steady and uniform flow (or drift), which is defined by its velocity $\vec{v} = \{v_{\parallel}, v_{\perp_1}, v_{\perp_2}\}$. This is the simplest type of flow, where $v_{\parallel}$ is the velocity component in the direction from the transmitter to the receiver while $v_{\perp_1}$ and $v_{\perp_2}$ are the components perpendicular to the line from the transmitter to the receiver. The motion of each molecule is considered independent of each other and affected by the diffusion coefficient D, which is constant and identical for all molecules.

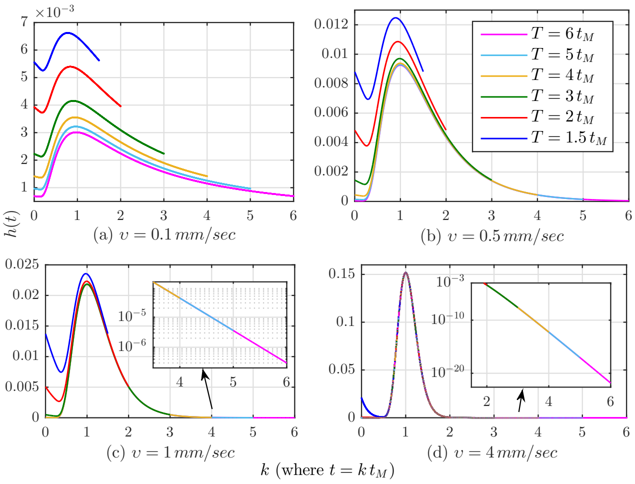

Fig. 2: $h(t)$ versus k for various values of υ and T.

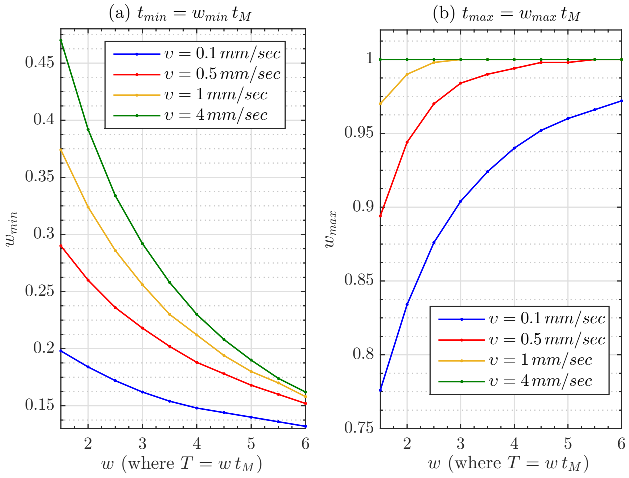

Fig. 3: t_{min} and t_{max} for various values of υ and T.

As depicted in Fig. 1, the receiver is a sphere with radius r and volume V, placed at a distance d from the transmitter. It works as an ideal passive observer that is perfectly synchronized with the transmitter and capable of identifying and counting the specific type of the information molecules entering its volume at any given time instant. Besides, noise sources are present in the communication environment emitting molecules of the same type as the information carrying ones. These noise molecules are mistaken as information molecules when found inside the receiver's volume, thus acting as additive external noise to the observations. In our study, we assume an additive noise Poisson random variable (RV) with constant mean. Finally, the system suffers from intersymbol interference (ISI) due to the lingering presence at the receiver of information molecules corresponding to a previous transmission. However, in contrast to [7], the presence of flow here mitigates the ISI by carrying molecules away from the receiver.

III. Theoretical Analysis

Due to their stochastic movement in the communication fluid, the information molecules reach the receiver in a probabilistic manner. Thus, their number within the receiver is a RV following Poisson distribution with a

TABLE I: Parameters of the considered MC system

D [m²/sec]	10^{-9}	$\upsilon_{\parallel} = \upsilon$ [mm/sec]	0.1, 0.5, 1 or 4
d [μm]	12	$\upsilon_{\perp_1}$,$\upsilon_{\perp_2}$ [mm/sec]	0
r [μm]	2	M	5000
I	10	μ_N	40 or 500

proper mean value [3]. The probability that an information molecule emitted at $t = 0$ lies within the receiver's volume V at t is [3]

$$h(t) = \frac{V}{(4\pi Dt)^{3/2}} \exp\left(-\frac{|\vec{r}_d|^2}{4Dt}\right), \tag{1}$$

where $|\vec{r}_d|^2 = (d - \upsilon_{\parallel} t)^2 + (\upsilon_{\perp} t)^2$ and $\upsilon_{\perp}^2 = \upsilon_{\perp_1}^2 + \upsilon_{\perp_2}^2$. Meanwhile, the probability that a molecule released at $t = 0$ is observed within the receiver, reaches its global maximum at

$$t_{\rm M} = \frac{-3 + \sqrt{9 + \frac{d^2\beta}{D}}}{\beta}, \tag{2}$$

where $\beta = (\upsilon_{\parallel}^2 + \upsilon_{\perp}^2)/D$.

In this paper, we assume that the receiver makes L observations at time instants given by $f(l) \in [0, T]$, where $l = 1, 2, .., L$. Therefore, if we denote with x_m the bit transmitted in the mth transmission, the number of received molecules corresponding to the lth sample taken within the mth transmission interval yields

$$N_{m,l} \sim \text{Pois}\left(\sum_{i=0}^{I} M x_{m-i} h(f(l) + iT) + \mu_N\right), \tag{3}$$

where μ_N stands for the mean value of the noise molecules observed per time sample and I is the number of previous transmissions taken into account, thus, considered as ISI. These observations or samples are summed up and, afterwards, the single threshold detector with an appropriate threshold value τ is employed in order to detect the transmitted bit as follows

$$\hat{x}_m = \begin{cases} 1, & \text{if } \sum_{l=1}^{L} N_{m,l} \geq \tau \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Utilizing the approximation of the Poisson distribution with a Gaussian distribution and an appropriate continuity correction, a closed-form expression for the mean error probability, P_e, can be derived as

$$P_e = \left(\frac{1}{2}\right)^{I+2} \sum_{q=1}^{2^I} \left[\text{erfc}\left(\frac{\sum_{l=1}^{L} \mu_l(q,1) - \tau + 0.5}{\sqrt{2\sum_{l=1}^{L} \mu_l(q,1)}}\right) + \text{erfc}\left(\frac{\tau - 0.5 - \sum_{l=1}^{L} \mu_l(q,0)}{\sqrt{2\sum_{l=1}^{L} \mu_l(q,0)}}\right) \right], \tag{5}$$

where $\text{erfc}(\cdot)$ is the complementary error function. The mean value $\mu_l(q, \text{x})$ can be computed as

$$\mu_l(q, \text{x}) = M\text{x}h(f(l)) + \sum_{j=1}^{I} [M\text{v}_j h(f(l) + jT)] + \mu_N, \tag{6}$$

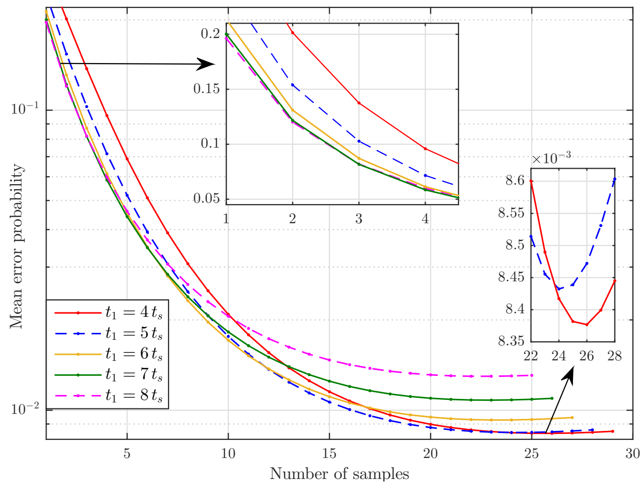

Fig. 4: P_e versus the number of samples for various values of the time when the sampling process starts at t_1, where $\upsilon = 0.1$ mm/sec, $T = 4\,t_{\mathrm{M}}$, $t_s = t_{\mathrm{M}}/8$, and $\mu_N = 40$.

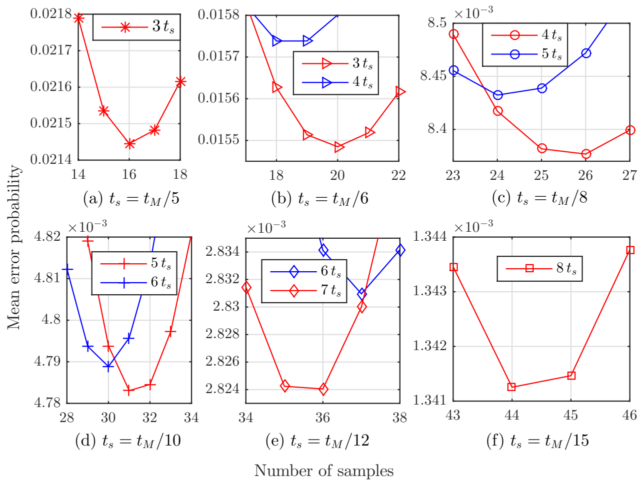

Fig. 5: P_e for various values of the sampling time t_s, where $\upsilon = 0.1$ mm/sec, $T = 4\,t_M$, and $\mu_N = 40$.

TABLE II: Optimal values of t_1 and t_2 in Fig. 5

case	(a)	(b)	(c)	(d)	(e)	(f)
t_1	$0.6\,t_{\mathrm{M}}$	$0.5\,t_{\mathrm{M}}$	$0.5\,t_{\mathrm{M}}$	$0.5\,t_{\mathrm{M}}$	$0.58\,t_{\mathrm{M}}$	$0.53\,t_{\mathrm{M}}$
t_2	$3.6\,t_{\mathrm{M}}$	$3.6\,t_{\mathrm{M}}$	$3.6\,t_{\mathrm{M}}$	$3.5\,t_{\mathrm{M}}$	$3.5\,t_{\mathrm{M}}$	$3.4\,t_{\mathrm{M}}$

where q corresponds to each of the 2^I possible bit sequences $\{\mathrm{v}_1, \mathrm{v}_2, .., \mathrm{v}_I\}$, which stand for the I previous transmissions and are taken into account as ISI.

IV. Simulations and Discussion

In the following, we assume that flow exists only towards the direction of the line from the transmitter to the receiver. Thus, the perpendicular components of the velocity, $\upsilon_{\perp_1}$ and $\upsilon_{\perp_2}$, are set equal to 0 and we refer to the parallel component $\upsilon_{\|}$ as υ for simplicity. The utilized system parameters are mentioned in Table I, while I is set equal to 10 to take into account almost all the experienced ISI. Finally, a sampling time t_s is considered while adjacent samples are equally spaced in time, i.e., $f(l) = l\,t_s$.

In Fig. 2, $h(t)$ is depicted against k, where $t = k\,t_{\mathrm{M}}$. Moreover, in Fig. 3, $t_{\min}$ corresponds to the local minimum of $h(t)$ for $t < t_{\mathrm{M}}$ and $t_{\max}$ to the global maximum. It can be inferred that $t_{\min}$ approaches the time when the ISI molecules become less than those corresponding to the current transmission. In the absence of ISI or when ISI is negligible, the maximum number of information molecules will be observed at $t_{\max} = t_{\mathrm{M}}$ given by (2). However, $t_{\max}$ does not equal t_M when ISI is dominant, namely for low values of the velocity and/or the symbol duration. As expected, $t_{\min}$ decreases and $t_{\max}$ increases with increasing T or velocity due to the decrease in ISI.

Interestingly, it can be inferred from (2) that as the velocity increases, t_{M} decreases, which is rational because flow facilitates the transmission. Therefore, it is expected that in the case of high velocity, the time interval where the sampling process occurs, i.e., $[t_1, t_2]$, is of smaller duration and, thus, t_1 and t_2 are closer to t_{M}. This has been verified in the simulations that follow.

In Fig. 4, the achievable error probability is depicted versus the number of samples for different considerations regarding the timing of the first sample, t_1. Interestingly, the error rate is minimized when $L_{\mathrm{opt}} = 26$ samples are collected while the first observation is made at $t_1 = 4\,t_s$, i.e., the sampling interval equals $[t_1, t_2] = [4t_s, 29t_s]$ with a space in time equal to t_s between adjacent samples. Moreover, it should be noted that if the receiver could collect only one sample, this should be at $t_1 = 8\,t_s$, which equals t_{M}.

In Fig. 5, the minimum error rate for various sampling time values can be observed. It can be easily inferred that as the sampling time decreases, the error performance enhances and the optimal number of samples increases. As can be seen from Table II, the time t_1 when the first sample should be collected is equal to or a bit higher than $t_{\mathrm{M}}/2$, while the time t_2 when the sampling process finishes is very close to $3.5\,t_{\mathrm{M}}$. Thus, t_1 and t_2 do not vary much with the sampling time t_s.

In Fig. 6, the minimum error rate for various symbol duration values can be observed. When the symbol duration

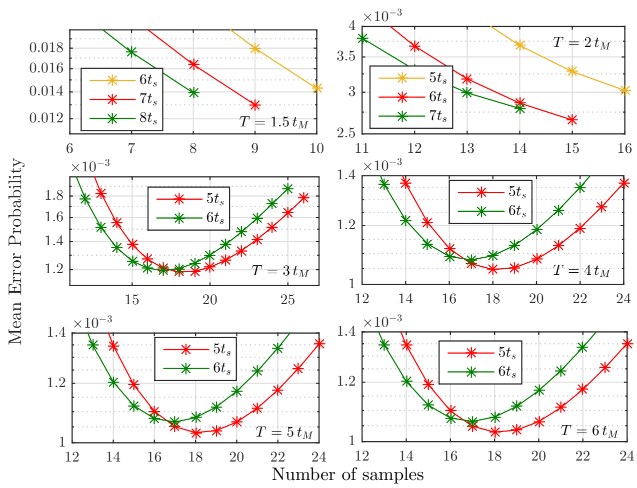

Fig. 6: P_e for various values of the symbol duration T, where $\upsilon = 0.5$ mm/sec, $t_s = t_M/10$, and $\mu_N = 500$.

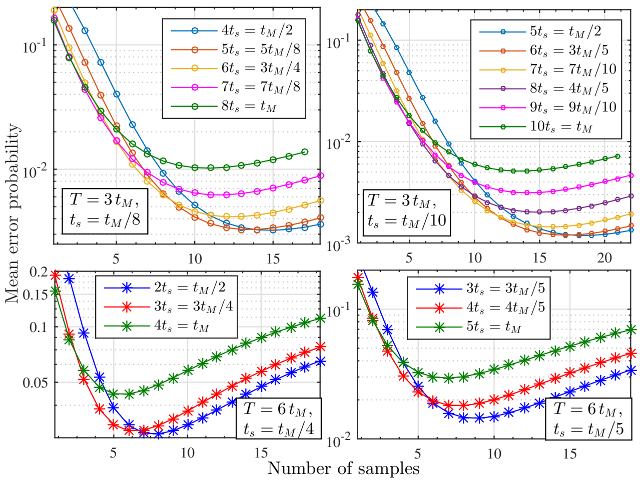

Fig. 7: P_e for various values of T, t_s and t_1, where $\upsilon = 0.5$ mm/sec and $\mu_N = 500$.

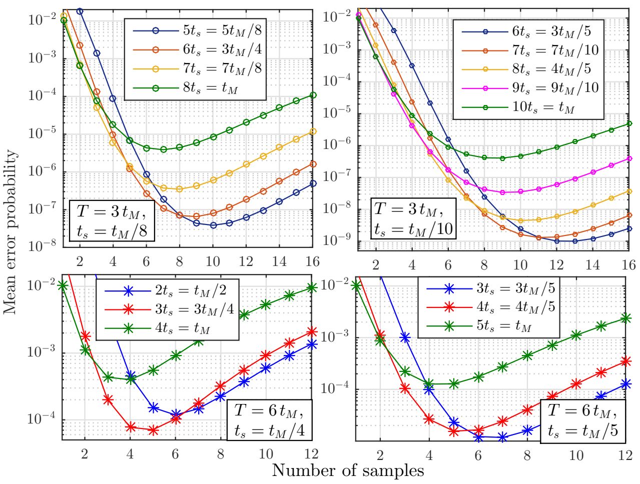

Fig. 8: P_e for various values of T, t_s and t_1, where $\upsilon = 1$ mm/sec and $\mu_N = 500$.

TABLE III: Optimal values of t_1 and t_2 in Figs. 7 and 8

Fig.	7	8
t_1	$[0.5, 0.625]\, t_{\mathrm{M},\upsilon=0.5}$	$[0.6, 0.75]\, t_{\mathrm{M},\upsilon=1}$
t_2	$[2.125, 2.25]\, t_{\mathrm{M},\upsilon=0.5}$	$[1.75, 1.8]\, t_{\mathrm{M},\upsilon=1}$

is small and, thus, the ISI high, the sampling process starts later than $t_\mathrm{M}/2$ and finishes at the end of the symbol duration, i.e., $t_2 = T$ for $T \leq 2\,t_\mathrm{M}$. However, for $T \geq 3\,t_\mathrm{M}$, the sampling process starts at $t_1 = t_\mathrm{M}/2$ and finishes at $t_2 = 2.2\,t_\mathrm{M}$, while the enhancement of the error performance with increasing T is small. Therefore, t_1 and t_2 depend on the velocity value and not on the symbol duration, as long as the later is high enough. Meanwhile, increasing the symbol duration above $4\,t_\mathrm{M}$ has been proven inefficient, as it results in a slight enhancement of the error performance, though at the cost of a detrimental decrease in the transmission rate.

Comparing Figs. 7 and 8, it can be deduced that higher velocity values enhance the error performance thanks to the decrease of the induced ISI. Besides, this results in higher transmission rate as t_M decreases with increasing velocity. Moreover, the optimal number of samples that should be collected is smaller for the case of $\upsilon = 1$ mm/sec, as obvious from Table III. Finally, it should be emphasized that, with increasing velocity, t_2 decreases significantly while t_1 increases slightly. This is in accordance with the observation made in Fig. 1, where the maximum value of $h(t)$ increases with velocity, and meanwhile the variance of t decreases around $t_{\max}$.

V. Conclusion

Diffusion-based MC systems where the information molecules are characterized by a composite diffusive-drift motion have been investigated thoroughly in terms of the optimal sampling time and number of observations at the receiver side that minimize the error probability. Interestingly, it has been proved that by optimizing both these parameters, the system performance is enhanced significantly. Particularly, the most interesting findings derived from our analysis can be summarized as follows:

1) The starting time, t_1, of the observation interval is very close to $0.5\,t_\mathrm{M}$ in all cases, where t_M is the critical time epoch given by (2).
2) The ending time, t_2, of the observation interval depends on the drift velocity, υ, of the molecules and decreases significantly with increasing υ.
3) Increasing the symbol duration, T, above a particular value, i.e., $4\,t_\mathrm{M}$ in our case, is inefficient, as it does not enhance the error performance significantly while it reduces the transmission rate.
4) Decreasing the sampling time, t_s, is advantageous in terms of the error performance.

An extension of this work can be considered in two directions; the investigation of the system model under study where the perpendicular components of the flow velocity are also present (i.e., non-trivial), and the derivation of closed-form expressions for t_1 and t_2.

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